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REMARKS ON THE SCHWARZSCHILD-MILNE MODEL OF THE OUTER LAYERS OF A STAR. II.

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(Communicated by Prof. E. A. Milne)

This note is a continuation of an earlier paper bearing the same title.* It was shown there that the exact solution $B(\tau)$ of Milne's problem is of the form

$$B(\tau) = \tfrac{3}{4}F(\tau + \phi(\tau)), \qquad \tfrac{1}{2} < \phi(\tau) < 1,$$

where πF denotes the net flux constant, and where $\phi(\tau)$ is the uniquely determined solution of the integral equation

$$\phi(\tau) = \tfrac{1}{2}\int_0^\infty \phi(t) Ei_1(|\tau - t|)dt + \tfrac{1}{2}Ei_3(\tau). \tag{1}$$

The functions $Ei_n(\tau)$ are defined by

$$Ei_n(\tau) = \int_0^\infty \frac{e^{-s\tau}}{s^n} ds, \qquad n \geqq 0.$$

Furthermore, it was proved that

$$\phi(0) = \frac{1}{\sqrt{3}}, \tag{2}$$

yielding the relation

$$T_e^4 = \frac{4}{\sqrt{3}} T_0^4$$

between the effective temperature and the boundary temperature.

* *M.N.*, **90**, 287, 1930. This paper is quoted as I. Its main results have independently been proved by Dr. M. Bronstein, whose notes appeared a little earlier than the author's paper. Cf. *Zeits. für Phys.*, **58**, 696, 1929, and **59**, 144, 1929.

In the following note we prove that $\phi(\tau)$ increases with increasing τ, a result which easily yields the following facts. We deduce that

$$0{\cdot}7083 < \tfrac{17}{24} < \phi(\infty) = \lim_{\tau=\infty} \phi(\tau) < \tfrac{5}{7} < 0{\cdot}7143. \tag{3}$$

The function $\phi(\tau)$ lies therefore in a narrow interval

$$0{\cdot}577 < \phi(\tau) < 0{\cdot}715, \qquad 0 \leqq \tau < \infty.$$

The well-known first approximation

$$\phi_1(\tau) = \tfrac{2}{3},$$

which has been used with so much success, is thus seen to be of considerable accuracy for most purposes. Narrow ranges can also be found for some other quantities.

§ 1. *Inequalities for $\phi(\infty)$ and other Quantities.*

As in I, it is convenient to use the symbols

$$(f, g) = \int_0^\infty f(t)g(t)dt$$

$$\Lambda(f) = \tfrac{1}{2}\int_0^\infty f(t)Ei_1(|\tau - t|)dt.$$

The symmetry of the kernel appears in the formula

$$(f, \Lambda(g)) = (g, \Lambda(f)). \tag{4}$$

The solution of (1) satisfies the equation

$$(\phi, Ei_2) = \tfrac{1}{3} \tag{5}$$

(formula (19) in I). The proof that $\phi(\tau)$ is an increasing function may be postponed until § 2. In setting

$$\delta(\tau) = \phi(\tau) - \phi(\infty) \tag{6}$$

we infer from

$$1 = \Lambda(1) + \tfrac{1}{2}Ei_2 \tag{7}$$

(formula (13) in I) that

$$\delta = \Lambda(\delta) + \tfrac{1}{2}Ei_3 - \tfrac{1}{2}\phi(\infty)Ei_2. \tag{8}$$

From the easily verifiable differentiation formula

$$\Lambda(f)' = \Lambda(f') + \tfrac{1}{2}f(0)E_1$$

and from (7) we obtain

$$\tau = \Lambda(t) - \tfrac{1}{2}Ei_3. \tag{9}$$

According to the formulæ

$$(t, Ei_n) = (1, Ei_{n+1}) = Ei_{n+2}(0) = \frac{1}{n+1} \tag{10}$$

we have from (8)

$$(t, \delta - \Lambda(\delta)) = \tfrac{1}{8} - \tfrac{1}{6}\phi(\infty).$$

From (4) and (9) the left-hand side is equal to

$$(\delta, t-\Lambda(t)) = -\tfrac{1}{2}(\delta, Ei_3) = -\tfrac{1}{2}(\phi, Ei_3)+\tfrac{1}{6}\phi(\infty),$$

thus yielding

$$\phi(\infty) = \tfrac{3}{2}(\phi, Ei_3)+\tfrac{3}{8}.* \tag{11}$$

According to (5), (11) may be written

$$\phi(\infty) = \tfrac{3}{2}(\phi, Ei_3-\tfrac{1}{2}Ei_2)+\tfrac{5}{8}. \tag{12}$$

We now make use of the inequalities

$$nEi_{n+1} \geqq mEi_{m+1}, \qquad \tau \geqq 0, \qquad n > m, \tag{13}$$

due to Dr. Ambarzumian.† From (12), (13) we obtain

$$\phi(\infty) < \tfrac{3}{2}\phi(\infty)(1, Ei_3-\tfrac{1}{2}Ei_2)+\tfrac{5}{8} = \tfrac{1}{8}\phi(\infty)+\tfrac{5}{8},$$

thus yielding $\phi(\infty) < \frac{5}{7}$. On the other hand, partial integration yields

$$(\phi, Ei_3) = \tfrac{1}{3}\phi(0)+(\phi', Ei_4) \tag{14}$$

and, since $\phi' \geqq 0$,

$$(\phi', Ei_4) > \tfrac{2}{3}(\phi', Ei_3) = \tfrac{2}{3}[-\tfrac{1}{2}\phi(0)+(\phi, Ei_2)] = \tfrac{2}{9}-\tfrac{1}{3}\phi(0). \tag{15}$$

On combining (14) and (15) we get

$$(\phi, Ei_3) > \tfrac{2}{9},$$

and according to (11),

$$\phi(\infty) > \tfrac{1}{3}+\tfrac{3}{8}, \textit{ i.e. } (3).$$

As to the law of darkening,

$$I(0, \theta) = \tfrac{3}{4}F\{\cos\theta+\int_0^\infty e^{-s}\phi(s\cos\theta)ds\} \tag{16}$$

(see I, p. 289), we infer that the integral decreases steadily from

$$\int_0^\infty e^{-s}\phi(s)ds \qquad \text{to} \qquad \phi(0) = \frac{1}{\sqrt{3}},$$

if θ increases from 0 to $\frac{\pi}{2}$. For $\theta = 0$, the value of the integral is (ϕ, e^{-t}). From

$$tEi_2(t) = e^{-t}-2Ei_3(t)$$

we get

$$(\phi', tEi_2) = (\phi', e^{-t}-2Ei_3(t)) = (\phi, e^{-t}-2Ei_2) = (\phi, e^{-t})-\tfrac{2}{3},$$

thus yielding ($\phi' \geqq 0$)

$$\tfrac{2}{3} < (\phi, e^{-t}) < \tfrac{2}{3}+(\phi(\infty)-\phi(0))\max tEi_2(t) < \tfrac{2}{3}+0{\cdot}023.$$

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* This formula, in a different form, was first derived by Dr. Bronstein, *M.N.*, **91**, 133, 1930. The numerical range given there for $\varphi(\infty)$ is much wider than (3).

† They follow from $Ei_{n+1} < Ei_n$ and from $nEi_{n+1} = e^{-\tau}-\tau Ei_n$. What we have denoted by Ei_n, Dr. Bronstein and Dr. Ambarzumian call Ei_{n-1}.

If θ runs from $\frac{\pi}{2}$ to 0 (from limb to centre), $I(0, \theta)$ lies first a little below the first approximation

$$I_1(0, \theta) = \tfrac{3}{4}F\{\cos\theta + \tfrac{2}{3}\}$$

(corresponding to $\phi_1(\tau) = \frac{3}{4}$) and rises then a little above it.

§ 2. *Proof that $\phi(\tau)$ Increases.*

The fact that $\phi(\tau)$ increases lies rather deep. It seems to be inaccessible by the classical comparison methods used so much in the theory of integral equations with positive kernel. We consider the class of all (also discontinuous) nowhere decreasing functions $\psi(\tau) > 0$ with the property

$$\psi(\tau) \geqq \Lambda(\psi) + \tfrac{1}{2}Ei_3(\tau), \qquad \tau \geqq 0. \tag{17}$$

Such functions ψ exist according to (7) and to $Ei_2 > Ei_3$. Let $u(\tau)$ be the lower bound of the values of all functions ψ at τ. Obviously $u(\tau)$ never decreases. From $\psi \geqq u$, $\tau \geqq 0$, we get $\psi \geqq \Lambda(u) + \frac{1}{2}Ei_3$. Since this inequality holds for any ψ of the above class, we infer

$$u(\tau) \geqq \Lambda(u) + \tfrac{1}{2}Ei_3(\tau), \qquad \tau \geqq 0\,; \qquad \frac{u(\tau_2) - u(\tau_1)}{\tau_2 - \tau_1} \geqq 0\,; \tag{18}$$

in other words, the above class of functions ψ contains a smallest one.

If we can prove that in the first inequality of (18) the $=$ sign applies everywhere, it follows at once that $u(\tau) \equiv \phi(\tau)$, *i.e.* that $\phi(\tau)$ never decreases. Let us call a point $\tau = \tau_0$ a regular point, if $u(\tau) > u(\tau_0)$ for all $\tau > \tau_0$. $\tau = \tau_0$ is, on the other hand, an irregular point, if τ_0 is the left end-point of a constancy interval of u, $u(\tau) = u(\tau_0)$, $\tau_0 < \tau < \tau_0 + \delta$. We now prove the following :—

Lemma.— $$u(\tau) = \Lambda(u) + \tfrac{1}{2}Ei_3(\tau) \tag{19}$$

holds at $\tau = 0$ and at all regular points.

Proof.—Let $\tau = \tau_0$ be a regular point, and let us suppose that

$$u(\tau) > \Lambda(u) + \tfrac{1}{2}Ei_3(\tau)$$

at $\tau = \tau_0$. It follows from reasons of continuity that in a certain finite interval (τ_0, τ_1), $\tau_0 < \tau_1$,

$$u(\tau) > \Lambda(u) + \tfrac{1}{2}Ei_3(\tau) + \epsilon, \qquad \tau_0 \leqq \tau \leqq \tau_1, \tag{20}$$

ϵ being a suitably chosen positive number. We consider then the auxiliary function

$$\begin{aligned} h(\tau) &= u(\tau) - u(\tau_0), && \tau \text{ in } (\tau_0, \tau_1), \\ h(\tau) &= 0, && \tau \text{ not in } (\tau_0, \tau_1). \end{aligned} \tag{21}$$

We note that

$$\Lambda(h) \geqq h, \qquad \tau \text{ not in } (\tau_0, \tau_1). \tag{22}$$

From (18) and (22) we now infer that the function $u(\tau) - \lambda h(\tau)$, λ being a positive constant, satisfies the inequality

$$u - \lambda h \geqq \Lambda(u - \lambda h) + \tfrac{1}{2}Ei_3 \tag{23}$$

outside the interval (τ_0, τ_1). In choosing $\lambda < 1$ so small that

$$|\lambda h - \Lambda(\lambda h)| = \lambda |h - \Lambda(h)| < \epsilon$$

in (τ_0, τ_1), we infer from (20) that (23) holds also inside (τ_0, τ_1), *i.e.* that (23) holds for all τ. On the other hand, from (21),

$$\begin{aligned} u - \lambda h &= u(\tau), && \tau \text{ not in } (\tau_0, \tau_1) \\ u - \lambda h &= (1-\lambda)u(\tau) + \lambda u(\tau_0), && \tau \text{ in } (\tau_0, \tau_1). \end{aligned} \tag{24}$$

Hence $u - \lambda h$ is a non-decreasing function, *i.e.* $u - \lambda h$ belongs to the above class of functions ψ. However, (24) shows that

$$\psi = u - \lambda h < u \text{ in } (\tau_0, \tau_1),$$

in contradiction to the definition of u as the lower limit of all ψ's. Hence (19) must be true for each regular point. As to the origin $\tau = 0$, the whole argument may be repeated, in using the auxiliary function

$$\begin{aligned} h(\tau) &= 0, && \tau > \delta \\ h(\tau) &= 1, && 0 < \tau < \delta. \end{aligned}$$

By means of a similar auxiliary function (where the left end-point of the interval is not necessarily the origin) we can infer that $u(\tau)$ is a continuous function.

So far we have made use only of the positivity of the kernel in (1). Functions $u(\tau)$ may be introduced in more general problems connected with integral equations having a positive kernel. We have now to take into account particular properties of our problem. We show that $u(\tau)$ has no irregular points. This proves, firstly, that $u(\tau)$ increases with increasing τ, and, secondly, according to the lemma, that $u(\tau)$ is a solution of (1).

The existence of an irregular point implies the existence of a constancy interval of u. We consider such an interval (if existing)

$$I : \tau' < \tau < \tau''$$

and we suppose it to be as large as possible. If $\tau' = 0$, (19) holds at $\tau = \tau'$ according to the lemma. If, on the other hand, $\tau' > 0$, an arbitrarily small left neighbourhood of τ' contains regular points. Hence, from continuity reasons, we infer that (19) holds at $\tau = \tau'$. In any case, (19) therefore holds at $\tau = \tau'$. Now we have

$$2\Lambda(f) = \int_0^\tau f(t) dEi_2(\tau - t) - \int_\tau^\infty f(t) dEi_2(t - \tau),$$

whence, by partial integration,

$$2\Lambda(f) = -f(0)Ei_2(\tau) + 2f(\tau) - \int_0^\tau Ei_2(\tau - t) df(t) + \int_\tau^\infty Ei_2(t - \tau) df(t).$$

(18) may therefore be written

$$\frac{Ei_3(\tau)}{Ei_2(\tau)} - u(0) - \int_0^\tau \frac{Ei_2(\tau - t)}{Ei_2(\tau)} du(t) + \int_\tau^\infty \frac{Ei_2(t - \tau)}{Ei_2(\tau)} du(t) \leqq 0.$$

Since u is constant in (τ', τ''), we have

$$\frac{Ei_3(\tau)}{Ei_2(\tau)} - u(0) - \int_0^{\tau'} \frac{Ei_2(\tau - t)}{Ei_2(\tau)} du(t) + \int_{\tau''}^{\infty} \frac{Ei_2(t - \tau)}{Ei_2(\tau)} du(t) \begin{cases} = 0, & \tau = \tau', \\ \leqq 0, & \tau' < \tau < \tau''. \end{cases} \quad (25)$$

The functions

$$\frac{Ei_{n+1}(\tau)}{Ei_n(\tau)}$$

are readily seen to increase with increasing τ. Furthermore, we have

$$\frac{d}{d\tau}\left(\frac{Ei_2(\tau - t)}{Ei_2(\tau)}\right) = \frac{E_2(\tau - t)}{E_2(\tau)}\left(\frac{E_1(\tau)}{E_2(\tau)} - \frac{E_1(\tau - t)}{E_2(\tau - t)}\right) < 0, \quad \tau > t > 0.$$

From $Ei_2' = -Ei_1$ we see that

$$\frac{Ei_2(t - \tau)}{Ei_2(\tau)} (\tau < t)$$

increases with increasing τ. According to $du(t) \geqq 0$, the left-hand side in (25) increases therefore with increasing τ, $\tau' \leqq \tau \leqq \tau''$, in contradiction to (25). Thus u cannot have any irregular points.

Cambridge :
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DIRAC'S EQUATION IN GENERAL RELATIVITY.

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I. *Introduction*

In a recent paper * Schroedinger has proposed a generalized form of Dirac's first order equation for the electron, which is valid in any Riemannian space of four dimensions. The new equation thus enables us to take into account, in the Quantum Theory, the effect of gravitational fields on the properties of the electron. The aim of the present investigation is to deduce from the new equation certain results, well known in the relativity theory, concerning the behaviour of the light emitted by atoms in gravitational fields which have not, hitherto, been obtainable from quantum mechanics.

The phenomena to which we refer are, firstly, the shift towards the red (compared with terrestrial measurements) of the spectral lines emitted by atoms in the strong gravitational field near the Sun or a star, and, secondly, the red-shift of the lines in the spectra of the extra-galactic nebulæ. The treatment of these two phenomena is not the same in general relativity. To account for the first, it is demonstrated that the period of any vibrating system, whose structure may be quite unknown but must be assumed invariable, is affected in a certain way when its position in a gravitational field is altered. If light is emitted by a vibrating system, it too must suffer a change of frequency

* E. Schroedinger, *Preuss. Akad. Wiss.*, Berlin, Ber. 11–12, pp. 105–128, 1932.

when the emitting agent is moved from one position to another. The amount can be predicted theoretically and is found to agree with experiment. The second phenomenon is dealt with in a different way : it is assumed to be due to a kind of Doppler effect produced by the expansion of space. Underlying this description of the phenomenon there is the further assumption that, whatever changes may be produced in the structure of an atom by the expansion of space as a whole, the frequency of the light it emits remains constant throughout all time.

Schroedinger has now, in effect, defined what is to be meant by an electron and an atom in a gravitational field. He has done this entirely by constructing an equation possessing certain properties of mathematical invariance and which reduces to Dirac's equation in Euclidean space. This procedure is familiar in the treatment of a similar problem, that of the "unified field-theory of gravitation and electromagnetic fields," where it has led to the construction of an endless series of different theories, none of which appears capable of giving any new results which could be compared with experiment. It has therefore seemed worth while to show that Schroedinger's new equation for the electron does actually give results, not deducible from the previous quantum theory, which are in accord with experiment and which can also be deduced from general relativity. We shall show in what follows that, for a hydrogen atom, (*a*) the intensity of the gravitational field affects the frequency of emitted light in the manner actually observed ; (*b*) the frequency is not affected by the expansion or contraction of the universe, although the latter does affect the energies of the levels.

II. *Schroedinger's Theory*

We give a short summary of Schroedinger's theory. In a space whose metric is

$$ds^2 = g_{00}dx_0^2 + 2g_{0i}dx_0dx_i + g_{ik}dx_idx_k, \tag{1}$$

where

$$i, k = 1, 2, 3 \quad \text{and} \quad x_0 = ict,$$

the g_{ik} are expressed by means of matrices γ_i such that

$$\gamma_i\gamma_k + \gamma_k\gamma_i = 2g_{ik}. \tag{2}$$

Applying an infinitesimal point transformation to (2), it can be shown that the γ_i must satisfy the set of partial differential equations

$$\frac{\partial\gamma_i}{\partial x_e} = \Gamma_{ie}{}^{\mu}\gamma_\mu + \Gamma_e\gamma_i - \gamma_i\Gamma_e, \tag{3}$$

where $\Gamma_{ie}{}^{\mu}$ are the Christoffel symbols for the metric (1), and Γ_e are certain new matrices, defined by (3) when the γ_i are given. The Γ_e are fundamental : for it follows from (3) and (2) that $\phi_e = \frac{i}{4}\,Spur\,\Gamma_e$ ($e = 0, 1, 2, 3$) is not determined by (3), and further that ϕ_e obeys the same equations as, and is therefore identified with, an electromagnetic potential.

The next step is to impose on the γ_i the condition that all sets, γ_i', given by

$$\gamma_i' = S^{-1}\gamma_i S, \tag{4}$$

where S is an arbitrary matrix, are equivalent. It then follows that the operator $\nabla_k = \frac{\partial}{\partial x_k} - \Gamma_k$ is an invariant for both point transformations and transformations of type (4). The last step is to generalize Dirac's equation in the form

$$\left(\alpha^0\frac{\partial}{\partial x_0} + \alpha^1\frac{\partial}{\partial x_1} + \alpha^2\frac{\partial}{\partial x_2} + \alpha^3\frac{\partial}{\partial x^3}\right)\psi = \frac{2\pi mc}{h}\psi,$$

where the α^s are any perpendicular set of 4-point matrices,* to

$$\gamma^k\left(\frac{\partial}{\partial x_k} - \Gamma_k\right)\psi = \frac{2\pi mc}{h}\psi, \tag{5}$$

where the γ^k are the contravariant forms † of the γ_i.

III. *Wave-equation in an Orthogonal Gravitational Field*

We shall now obtain the explicit form of the equation (5) applicable to any gravitational field whose metric is orthogonal and can be expressed by means of isotropic space co-ordinates in the form :

$$-ds^2 = \zeta^2 dx_0^2 + \alpha^2\mu^2(dx_1^2 + dx_2^2 + dx_3^2), \tag{6}$$

where ζ^2, μ^2 are functions of the space co-ordinates (x_1, x_2, x_3) alone and α^2 is a function of $x_0(=ict)$ alone.

We note that the Schwarzschild metric for the field around the Sun, in isotropic co-ordinates,‡ is obtained from (6) by putting

$$\zeta^2 = \frac{(1 - M/2r)^2}{(1 + M/2r)^2}, \qquad \mu^2 = \left(1 + \frac{M}{2r}\right)^4, \qquad \alpha = 1, \qquad r^2 = x_1^2 + x_2^2 + x_3^2, \tag{7}$$

whilst the metric of the "expanding universe" is the particular case of (6) for which

$$\zeta^2 = 1, \qquad \mu^2 = \frac{1}{(1 + r^2/4R^2)^2}, \qquad r^2 = x_1^2 + x_2^2 + x_3^2 \tag{8}$$

and α is a function of x_0.

The determination of the matrices γ_i for an orthogonal metric such as (6) is immediate. If E_0, E_1, E_2, E_3 is any set of perpendicular 4-point matrices, we have from (2)

$$\gamma_0 = \zeta E_0, \qquad \gamma_i = \alpha\mu E_i, \qquad i = 1, 2, 3. \tag{9}$$

With these values of the γ_i we have now to calculate the matrices Γ_e.

1. *The Matrices* Γ_e.—In order to solve the equations (3) we require the Christoffel symbols for the metric (6). If we denote differentiation of ζ, μ

* A. S. Eddington, *Proc. Roy. Soc.*, A, **121**, 524–542, 1928.

† Obtained by the usual rule, $\gamma^k = g^{ki}\gamma_i$.

‡ A. S. Eddington, *Math. Th. of Relativity*, 2nd ed., 1924, § 43.

by x_r ($r = 1, 2, 3$), by the symbols ζ_r, μ_r, and differentiation by x_0 by a dot, we find that the non-zero Christoffel symbols are the following (there is no summation) :—

$$\left.\begin{aligned} \{rr, r\} &= \frac{\mu_r}{\mu}, \qquad \{00, r\} = -\frac{\zeta\zeta_r}{\alpha^2\mu^2}, \qquad \{rr, 0\} = -\frac{\alpha\dot{\alpha}\mu^2}{\zeta^2} \\ \{0r, r\} &= \frac{\dot{\alpha}}{\alpha}, \qquad \{r0, 0\} = \frac{\zeta_r}{\zeta}, \qquad \{rr, s\} = -\frac{\mu_s}{\mu} \\ & \qquad\qquad \{sr, r\} = \frac{\mu_s}{\mu}, \end{aligned}\right\} \tag{10}$$

where r, s denote different suffixes which can take the values 1, 2, 3.

We can now write out the equations (3). We obtain the following set of 16 equations :—

$$\left.\begin{aligned} \Gamma_0 E_0 - E_0\Gamma_0 &= \sum_{s=1}^{3} \zeta_s E_s \\ E_r\Gamma_0 - \Gamma_0 E_r &= \frac{\zeta_r E_0}{\alpha\mu} \qquad r = 1, 2, 3 \end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned} \frac{\dot{\alpha}\mu}{\zeta}E_0 + \frac{\mu_2}{\mu}E_2 + \frac{\mu_3}{\mu}E_3 &= \Gamma_1 E_1 - E_1\Gamma_1, \\ \frac{\mu_2}{\mu}E_1 = E_2\Gamma_1 - \Gamma_1 E_2, \qquad \frac{\mu_3}{\mu}E_1 &= E_3\Gamma_1 - \Gamma_1 E_3, \\ \frac{\dot{\alpha}\mu}{\zeta}E_1 &= E_0\Gamma_1 - \Gamma_1 E_0, \end{aligned}\right\} \tag{12}$$

with eight more equations, giving Γ_2, Γ_3, whose construction is obvious from (12).

The solution of this set of equations is very easily obtained by using the properties * of perpendicular matrices, viz.

$$E_p^2 = 1, \qquad E_pE_q + E_qE_p = 0, \qquad E_p(E_qE_p) - (E_qE_p)E_p = -2E_q \tag{13}$$

for p, $q = 0$, 1, 2, 3 and p, q different suffixes.

We obtain in this way

$$\left.\begin{aligned} \Gamma_0 &= \frac{1}{2\alpha\mu}\left(\sum_{s=1}^{3}\zeta_s E_s\right)E_0 - i\phi_0 \\ \Gamma_1 &= \tfrac{1}{2}\left(\frac{\dot{\alpha}\mu}{\zeta}E_0 + \frac{\mu_2}{\mu}E_2 + \frac{\mu_3}{\mu}E_3\right)E_1 - i\phi_1 \\ \Gamma_2 &= \tfrac{1}{2}\left(\frac{\dot{\alpha}\mu}{\zeta}E_0 + \frac{\mu_1}{\mu}E_1 + \frac{\mu_3}{\mu}E_3\right)E_2 - i\phi_2 \\ \Gamma_3 &= \tfrac{1}{2}\left(\frac{\dot{\alpha}\mu}{\zeta}E_0 + \frac{\mu_1 E_1}{\mu} + \frac{\mu_2 E_2}{\mu}\right)E_3 - i\phi_3 \end{aligned}\right\}. \tag{14}$$

The quantities $(\phi_0, \phi_1, \phi_2, \phi_3)$ are arbitrary functions of (x_0, x_1, x_2, x_3) which

* A. S. Eddington, *Proc. Roy. Soc.*, A, **121**, 524–542, 1928. Equations (20) and (34).

are identified by Schroedinger with the components of the electromagnetic potential vector.

2. *Schroedinger's Equation.*—It remains now to insert the values of the Γ_e given by (14), and those of the γ^k deduced from (9), into the equation (5). If we do this and employ the results (13), we get

$$\left\{\frac{1}{\zeta}\left(\frac{\partial}{\partial x_0}+\frac{3}{2}\frac{\dot{a}}{a}+i\phi_0\right)E_0+\frac{1}{a\mu}\sum_{s=1}^{3}\left(\frac{\partial}{\partial x_s}+\frac{\zeta_s}{2\zeta}+\frac{\mu_s}{\mu}+i\phi_s\right)E_s\right\}\psi=\frac{2\pi mc}{h}\psi. \quad (15)$$

This, then, is the modified form of Dirac's equation for the electron in gravitational fields of type (6). To obtain numerical values we have to identify (E_0, E_1, E_2, E_3) with some set of perpendicular 4-point matrices. We take *

$$\left.\begin{aligned} E_0&=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}, \quad E_1=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\1&0&0&0\\0&-1&0&0\end{pmatrix}, \quad E_2=i\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&-1&0&0\\1&0&0&0\end{pmatrix}\\ E_3&=\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}\end{aligned}\right\}. \quad (16)$$

In what follows, we shall only be interested in those cases for which $\phi_1=\phi_2=\phi_3=0$ and $\phi_0\neq 0$. If we write

$$T_0\equiv\frac{1}{\zeta}\left(\frac{\partial}{\delta x_0}+\frac{3}{2}\frac{\dot{a}}{a}+i\phi_0\right), \quad T_s\equiv\frac{1}{a\mu}\left(\frac{\partial}{\partial x_s}+\frac{\zeta_s}{2\zeta}+\frac{\mu_s}{\mu}\right), \text{ and } \quad a\equiv\frac{2\pi mc}{h},$$

then the matrix equation (15) reduces, in virtue of (16), to the four first-order differential equations :

$$(T_0-a)\psi_1-(iT_2-T_3)\psi_0+T_1\psi_3=0, \quad (17)$$

$$(T_0-a)\psi_2+(iT_2+T_3)\psi_3-T_1\psi_0=0, \quad (18)$$

$$(T_0+a)\psi_3+(iT_2-T_3)\psi_2-T_1\psi_1=0, \quad (19)$$

$$(T_0+a)\psi_0-(iT_2+T_3)\psi_1+T_1\psi_2=0. \quad (20)$$

IV. *The Energy Levels of the Hydrogen Atom*

We shall now solve the equations (17) to (20) for the case corresponding to a single electron moving in an "orbit" under the attraction of the nuclear charge. Suppose that the nucleus has the co-ordinates (x_1^0, x_2^0, x_3^0) and let the co-ordinates of the electron be (x_1, x_2, x_3), where

$$x_1=x+x_1^0, \qquad x_2=y+x_2^0, \qquad x_3=z+x_3^0 \quad (21)$$

so that (x, y, z) are the co-ordinates of the electron relative to the nucleus.

* In Eddington's notation (*loc. cit.*, p. 529) these matrices are $(E_0, E_1, E_2, E_3)=(D_\alpha, S_\beta D_\beta, iS_\gamma D_\beta, S_\gamma)$.

Now it is known from the relativity theory that the functions ζ, μ, in all cases of physical interest, differ from unity by small quantities of the first order, whilst the functions $\left(\frac{\zeta_r}{2\zeta}+\frac{\mu_r}{\mu}\right)$ are of the second order of small quantities. From this it follows, since (x, y, z) are always small from the macroscopic point of view, that in the equations (17) to (20) we can treat ζ, μ as constants, with the values they have at the point (x_1^0, x_2^0, x_3^0) and neglect the terms $\left(\frac{\zeta_r}{2\zeta}+\frac{\mu_r}{\mu}\right)$ altogether. We shall thus obtain a first approximation for the effect of the gravitational field on the properties of the electron. We write

$$\zeta = 1+\epsilon, \qquad \mu = 1+\eta, \tag{22}$$

where ϵ, η are small constants whose squares we neglect. We shall further suppose that the rate of variation of α with the time is so slow compared with the period of the electron in its "orbit" that α and $\dot{\alpha}$ can be treated as constants in the equations (17) to (20).

It remains to discuss the form of ϕ_0 appropriate to the problem. We note that the spatial interval (an invariant) between two points whose co-ordinates are (x_1, x_2, x_3) and $(x_1+dx_1, x_2+dx_2, x_3+dx_3)$ is, by (6),

$$ids = \alpha\mu\sqrt{(dx_1^2+dx_2^2+dx_3^2)},$$

wherein μ has the value it has at the point (x_1, x_2, x_3). Remembering that (x, y, z) are always small, we can say that the spatial interval from the nucleus to the electron is

$$ids = \alpha\mu(x_1^0, x_2^0, x_3^0)r = \alpha(1+\eta)r,$$

where

$$r = \sqrt{(x^2+y^2+z^2)}.$$

We now know that, in Euclidean space, the appropriate value of ϕ_0 would be $\phi_0 = \frac{2\pi}{h}\frac{e^2}{c}\frac{1}{d}$, where d is the spatial interval between electron and nucleus. Hence, in our more general system of co-ordinates, we have

$$\phi_0 = \frac{2\pi}{h}\frac{e^2}{c}\frac{1}{\alpha(1+\eta)r}.$$

To find the energy levels we put, as usual,

$$\psi_r = e^{\frac{2\pi}{h}\frac{W}{c}x_0}\psi_r' \quad (r = 0, 1, 2, 3),$$

where W is the energy of one of the levels and ψ_r' is a function of (x, y, z) alone. If now we write

$$A_0 = \frac{2\pi}{h}\left(\frac{W}{c}+\frac{ie^2}{c\alpha(1+\eta)r}\right)+\frac{3}{2}\frac{\dot{\alpha}}{\alpha} \tag{23}$$

the equations (17) to (20) become (dropping the dashes for the ψ_r), using (21),

$$\{A_0(1-\epsilon)-a\}\psi_1-\frac{1-\eta}{\alpha}\left(i\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)\psi_0+\frac{1-\eta}{\alpha}\frac{\partial\psi_3}{\partial x}=0, \tag{17'}$$

$$\{A_0(1-\epsilon)-a\}\psi_2+\frac{1-\eta}{\alpha}\left(i\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\psi_3-\frac{1-\eta}{\alpha}\frac{\partial\psi_0}{\partial x}=0, \tag{18'}$$

$$\{A_0(1-\epsilon)+a\}\psi_3+\frac{1-\eta}{\alpha}\left(i\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)\psi_2-\frac{1-\eta}{\alpha}\frac{\partial\psi_1}{\partial x}=0, \tag{19'}$$

$$\{A_0(1-\epsilon)+a\}\psi_0-\frac{1-\eta}{\alpha}\left(i\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\psi_1+\frac{1-\eta}{\alpha}\frac{\partial\psi_2}{\partial x}=0. \tag{20'}$$

The method of solution we adopt is that given by C. G. Darwin * in terms of associated Legendre functions, P_k^u, where

$$\left.\begin{aligned} &P_k^u=(k-u)!\sin^u\theta\left(\frac{d}{d\cos\theta}\right)^{k+u}\frac{(\cos^2\theta-1)^k}{2^k k!}e^{iux} \\ &\text{and} \\ &\tan\chi=\frac{y}{z},\qquad \tan\theta=\frac{\sqrt{(y^2+z^2)}}{x}. \end{aligned}\right\} \tag{24}$$

We do not give the details of the calculation, which are to be found in Darwin's paper. The substitution

$$\psi_1=F(r)P_{k+1}^{u-1},\qquad \psi_3=(k+u)G(r)P_k^{u-1},$$
$$\psi_2=F(r)P_{k+1}^{u},\qquad \psi_4=-(k-u+1)G(r)P_k^{u},$$

is found to reduce † the four equations (17') to (20') to the two equations (a dash denoting differentiation with respect to r):

$$\{A_0(1-\epsilon)-a\}F-\frac{1-\eta}{\alpha}\left(G'-k\frac{G}{r}\right)=0, \tag{25}$$

$$\{A_0(1-\epsilon)+a\}G-\frac{(1-\eta)}{\alpha}\left(F'+\frac{(k+2)F}{r}\right)=0, \tag{26}$$

which serve to determine the unknown functions $F(r)$, $G(r)$. Multiplying the equations (25) and (26) by $\alpha(1+\eta)$ and putting

$$A^2=\alpha\left[\frac{2\pi}{h}\left\{mc+\frac{W}{c}(1-\epsilon)\right\}+\frac{3\dot{\alpha}}{\alpha}(1-\epsilon)\right](1+n), \tag{27}$$

$$B^2=\alpha\left[\frac{2\pi}{h}\left\{mc-\frac{W}{c}(1-\epsilon)\right\}-\frac{3\dot{\alpha}}{\alpha}(1-\epsilon)\right], \tag{28}$$

$$\bar{\gamma}=\frac{2\pi}{h}i\frac{e^2}{c}, \tag{29}$$

we obtain finally the equations

$$\left(A^2+\frac{\bar{\gamma}}{r}\right)G-\left(F'+\frac{(k+2)F}{r}\right)=0, \tag{30}$$

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* *Proc. Roy. Soc.*, A, **118**, 654, 1927.

† This reduction would not have been possible had the terms in $\left(\frac{\zeta_r}{2\zeta}+\frac{\mu_r}{\mu}\right)$ been retained in (17') to (20').

$$\left(B^2-\frac{\bar{\gamma}}{r}\right)F+\left(G'-\frac{k}{r}G\right)=0. \tag{31}$$

The calculation now follows Darwin's almost exactly except for the occurrence of i in certain terms. We have, in fact, if

$$F(r)=e^{-\lambda r}(a_0 r^{\beta}+a_1 r^{\beta-1}+\ \ldots), \tag{32}$$

$$G(r)=e^{-\lambda r}(b_0 r^{\beta}+b_1 r^{\beta-1}+\ \ldots), \tag{33}$$

then

$$\lambda=iAB, \qquad \beta=-1+\frac{i\bar{\gamma}(A^2-B^2)}{2AB}, \tag{34}$$

$$a_s=c_s\{A(\beta-k-s)+i\bar{\gamma}B\}, \qquad b_s=c_s\{A\bar{\gamma}+iB(\beta-s+k+2)\}, \tag{35}$$

where

$$\begin{aligned}2c_{s+1}(s+1)iAB&=c_s\{\bar{\gamma}^2-(\beta-s-k)(\beta-s+k+2)\}\\&=-c_s\{(s-\beta-1)^2-k_0{}^2\}\end{aligned} \tag{36}$$

and

$$k_0{}^2=(k+1)^2+\bar{\gamma}^2. \tag{37}$$

These results are obtained by equating to zero the successive powers of r when the series (32), (33) are introduced into (30) and (31).

Since the solution for F and G must be finite at $r=0$, the two series must terminate for some value of $s=n$ such that $\beta-n\geqslant 0$. From (36) we have that n is given by

$$n=\beta+1-k_0, \tag{38}$$

since this value of n makes c_{n+1}, c_{n+2}, etc., zero. Also since

$$\beta-n=k_0-1=\sqrt{\{(k+1)^2+\bar{\gamma}^2\}}-1, \quad \text{and} \quad (k+1)^2\ (>0)$$

is an integer whilst $\bar{\gamma}^2$ is a small negative constant, we have $\beta-n\geqslant 0$.

Equations (38) and (34) give the equation determining the energy-levels, viz. :

$$n+k_0=\frac{i\bar{\gamma}(A^2-B^2)}{2AB},$$

which may be written as a quadratic equation in W/c by the use of (27), (28). We have, in fact,

$$\frac{m^2c^2(1+2\epsilon)}{1+\dfrac{(i\bar{\gamma})^2}{(k_0+n)^2}}-\frac{9h^2}{4\pi^2}\frac{\dot{a}^2}{a^2}-\frac{W}{c}\frac{h}{\pi}\frac{3\dot{a}}{a}-\frac{W^2}{c^2}=0,$$

the solution of which is

$$\frac{W}{c}=\frac{mc(1+\epsilon)}{\left\{1+\dfrac{\gamma^2}{(n+\sqrt{(k+1)^2-\gamma^2})^2}\right\}^{\frac{1}{2}}}-\frac{3\dot{a}}{a}\frac{h}{2\pi}, \tag{39}$$

where $\gamma^2=\left(\dfrac{2\pi}{h}\right)^2\dfrac{e^4}{c^2}$, the fine-structure constant.

From this result it follows that the frequency, ν, of a line corresponding

to a transition from a state of total quantum number $N_1 = k_0 + n_1$ to another of number $N_2 = k_0 + n_2$ ($N_2 > N_1$) is given approximately by

$$h\nu = \frac{mc^2}{2}\gamma^2\left(\frac{1}{N_1^2} - \frac{1}{N_2^2}\right)(1+\epsilon). \tag{40}$$

We deduce the following results from (39) and (40):

(1) For the Schwarzschild metric (7),

$$1+\epsilon = 1 - M/R, \qquad \alpha = 1, \qquad \dot{\alpha} = 0, \qquad R = \sqrt{\{(x_1^0)^2 + (x_2^0)^2 + (x_3^0)^2\}}. \tag{41}$$

R is the distance (in cms.) from the centre of the Sun to the nucleus of the atom, M is the mass of the Sun in cms. Hence the ratio of the frequencies of a given line emitted on the Sun (ν_S) and on Earth (ν_E), respectively, is

$$\frac{\nu_S}{\nu_E} = 1 - \frac{M}{R},$$

since $\epsilon = 0$ at the distance of the Earth from the Sun. This is the well-known relativity result.

(2) For the "expanding universe" (8), we have

$$\epsilon = 0, \ \alpha \text{ and } \dot{\alpha} \text{ not zero.}$$

Hence, by (40), the frequency of the light emitted by an atom is unaffected by the expansion or contraction of the universe. Hence any observed reddening of the light from the nebulæ must be entirely due to some kind of Doppler effect, as is assumed in general relativity. We note also that the energy of each level is not constant, but decreases as the rate of expansion increases.

Lastly, we must verify that the terms involving $\left(\frac{\zeta_r}{2\zeta} + \frac{\mu_r}{\mu}\right)$, which we neglected in the equations (17′) to (20′), are actually negligibly small for (7) and (8). We find, for (7),

$$\frac{\zeta_r}{2\zeta} + \frac{\mu_r}{\mu} = -\frac{Mx_r^0}{2R^3}\frac{\left(1 - \frac{M}{R}\right)}{\left(1 - \frac{M^2}{4R^2}\right)},$$

neglecting terms involving (x, y, z). Since R here denotes the radius of the Sun, we see that the terms neglected were of the order of M/R^2 and were therefore negligibly small compared with those in M/R. Again, for the case (8),

$$\frac{\zeta_r}{2\zeta} + \frac{\mu_r}{\mu} = -\frac{x_r^0}{4R^2(1 + r_0^2/R^2)}.$$

Here $r_0^2 = \{(x_1^0)^2 + (x_2^0)^2 + (x_3^0)^2\}$ and R is the initial radius of the universe. Since, even for the most distant nebulæ, $x_r^0 < R$ and $R = 10^{27}$ cms., we were again justified in the approximation we adopted.

We conclude that Schroedinger's new wave-equation gives the properties which are ascribed to atoms in general relativity theory and which have

been verified experimentally. Nevertheless, it must be remarked that the new equation involves a number of terms of the second order (which we have neglected) which would give rise to perturbations of the atom not derivable by the method of general relativity. Unfortunately these terms are so very small as to preclude any hope of their giving rise to observable effects. To this extent, therefore, Schroedinger's equation must remain unverified.

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THE COEXISTENCE OF STELLAR AND INTERSTELLAR CALCIUM LINES IN THE MASSIVE B9 STAR H.D. 698.

J. A. Pearce, M.A., Ph.D. (Plate 16.)

(Communicated by Sir Arthur Eddington, F.R.S.)

I. *The Interstellar Cloud*

The theory of the nature and physical properties of the diffuse matter in interstellar space, as proposed by Eddington * in the Bakerian Lecture before the Royal Society, 1926, readily accounted for several phenomena connected with the so-called "stationary" lines of calcium and sodium. Thus the widespread distribution of the calcium cloud, relatively at rest with respect to the stars, as observed by Slipher † in 1909 from widely distributed Class B stars, and independently observed by Plaskett ‡ in 1924 from his observations of the Class O stars, naturally follows from the theory that this interstellar matter is uniformly distributed throughout the galactic plane. The observation of Young § that the "stationary" H and K lines were not associated with stars of lower temperature than Class B3 was readily explained by the fact that only the highly luminous stars of Classes O and B, whose absolute magnitudes exceeded $-2^{m}\cdot0$, had been observed at distances sufficient to produce the interstellar lines.

The theory accounted for the presence of ionised atoms of calcium and sodium in space. The earlier explanation that these atoms were ionised by the excitation of neighbouring high-temperature stars was now abandoned in favour of the view that the ionisation was produced by the general stellar radiation. Under the existing conditions of a density of the order of 10^{-24} gm./cm.^{3} and a temperature of 10,000° K. most of the calcium present would be doubly ionised, and sodium singly ionised, leaving a sufficient amount of neutral sodium and singly ionised calcium to cause the observed absorption.

The theory, however, required the confirmation of several points before its complete acceptance. Observational evidence of a quantitative nature

* *Proc. Roy. Soc.*, **111**, 424, 1926.
† *Low. Obs. Bull.*, **2**, 1, 1909.
‡ *Publ. D.A.O.*, **2**, 335, 1924.
§ *Ibid.*, **1**, 219, 1920.

was necessary to show the uniform distribution of the interstellar matter. The non-occurrence of the interstellar lines in the spectra of late-type stars was a second objection. Thirdly, the anomalous strength of the interstellar D lines of sodium required explanation, for according to Eddington there would only be one part in two million of neutral sodium available for the absorption of the interstellar D lines.

The first of these objections was partially removed by the investigations of Struve,* who showed that the intensities of the interstellar lines increased with the distances of the stars. Further confirmation was obtained by Gerasimovič and Struve,† who showed that the calcium cloud was rotating about the distant centre in Sagittarius, the uniform distribution being evident from an approximate two-to-one correspondence between the distances of the stars and cloud. A more complete discussion of "The Motions and Distribution of Interstellar Matter" was made by J. S. Plaskett and the writer ‡ in 1929 from their extensive material of the radial velocities of the O- and B-type stars. A general solution was first made for the solar motion, the K term and the galactic rotation of the interstellar cloud from the radial velocities of the K lines appearing in 261 stars of Classes O to B2, uniformly distributed in the galactic zone, approximately $l = 350°$ to $190°$, $b = 0°$ to $\pm 20°$. The analysis showed that the calcium cloud was relatively at rest with regard to the stars, having a K term of zero and a solar motion of 19·9 km./sec. directed towards an apex 20° away from the normal position, the discrepancy being due to the unsymmetrical distribution of the stars. Further, like the stars, the calcium cloud was in rotation about the distant galactic centre in the direction of Sagittarius.

A second solution for the elements of the galactic rotation, assuming a normal solar motion and a zero K term, gave the longitude of the galactic centre as $331°{\cdot}7 \pm 5°{\cdot}7$ coincident with the centre of the globular cluster system, while the rotational term of $\bar{r}A = 7{\cdot}9 \pm 0{\cdot}8$ km./sec. denoted a mean solar distance of approximately 465 parsecs for the cloud. Simultaneous solutions for the distances of the stars and the cloud, dividing the stars into (*a*) five groups according to apparent magnitudes, and (*b*) five groups according to the intensities of the K lines, resulted in a remarkable relationship of a two-to-one correspondence, the average distances of the stars being almost exactly double those of the clouds. When it was shown that this relationship held for star groups varying from 600 to 1600 parsecs in distance, no reasonable doubt remained regarding Eddington's hypothesis of an approximately uniform distribution of the cosmic matter.

A further research by the authors § from all available material, including 308 stars and 6 novæ showing interstellar sodium and calcium lines, has been recently completed. The analyses of the radial velocities confirm and amplify the findings of the preliminary investigation. A new method of estimating the intensities of the K lines was developed, and a linear relationship expressing the distances of the stars in terms of the intensities of the K lines established. The mean parallaxes of the several groups of stars

* *Ap. J.*, 65, 163, 1927 ; 66, 353, 1928.
† *Ibid.*, 69, 7, 1929.
‡ *M.N.*, 90, 243, 1930.
§ *Publ. D.A.O.*, 5, No. 3, in press.

were determined from proper motions.* These parallaxes were used with care to derive a value of the differential rotational constant $A = 0{\cdot}017$ km./sec. per parsec from the Sun. The investigation furnishes definite evidence in favour of both the rotation of the galaxy and the uniform distribution of the diffused matter throughout the region inhabited by the Classes O to B2 stars.

The second objection, "the non-appearance of the interstellar lines in the spectra of stars later than Class B5," has now been satisfactorily removed by the study of the massive binary system H.D. 698, of spectral type B9sek, discussed in this paper. The survey of the interstellar D lines now being made at the Mount Wilson Observatory by Dr. P. W. Merrill will supply the accurate and extensive observations on sodium which are urgently needed to explain the anomalous intensities of these lines and establish the theory for interstellar sodium.

II. *The Coexistence of Stellar and Interstellar Calcium Lines in the Spectra of the Stars of Classes* B2 *to* B5

It follows from the theory of a uniform distribution of the cosmic cloud that the interstellar calcium and sodium lines should be superimposed upon all stellar spectra as soon as the stars are at sufficient distances to show the interstellar absorption. They have been observed in practically all the O5 to B2 stars fainter than $5^{m}{\cdot}0$ and in many spectra of the faint B3 stars, but have not been observed in stars later than Class B5. The absence of the stellar calcium lines in the spectra of the O5 to B2 stars makes the identification of the interstellar calcium lines in these types certain. Stellar calcium makes its marginal appearance in Class B2, slowly increases in intensity through the later divisions of Class B, continually increases throughout Classes A, F and G, reaching a maximum intensity in Class M. In spectral types B3 to B5 the intensity of the stellar K line is comparable with that of the interstellar K line, so that for these subdivisions of Class B the coexistence of these two calcium lines of different origin is first observed. For the bright B3 to B5 stars nearer than 300 parsecs the measures of the faint K line show that it is entirely stellar in origin, but for the fainter B3 to B5 stars the measured velocities of the line indicate a blended origin whether the stars be spectroscopic binaries or possess a constant velocity.

The coexistence of stationary and oscillating K lines among the early Class B spectroscopic binaries has been known for many years. While suspected and reported for several stars, there are only 3 stars for which the coexistence has definitely been shown. The first case detected was that of H.D. 23180, *o* Persei, type B2k, a spectroscopic binary investigated by Jordan † in which both spectra appear and oscillate with large amplitudes. In addition to strong narrow H and K lines of constant velocity, Jordan observed a weak stellar K line oscillating with the same amplitude and in

* Unpublished proper motions for the faint O to B5 stars has been kindly placed at our disposal by the Director of the Dudley Observatory.

† *A.O.*, **2**, 63, 1910; **3**, 195, 1916.

the same period as the hydrogen and helium lines of the primary star. Somewhat similar results were found by Boothroyd * for the double-lined binary H.D. 218440, type B3k. In addition to a fairly strong K line of constant position, he found the presence of oscillating stellar K lines for both stars upon four spectrograms, these plates thus showing three components for the K line and giving indisputable evidence of the coexistence of stellar and interstellar calcium. This star has recently been observed by the writer,† who has determined the mass ratio of the two components. The three K lines appear on many of the spectrograms taken at the time of maximum separation of the two components.

The third case is that of the eclipsing variable U Ophiuchi, type B5nk, the orbit of which was investigated by Plaskett.‡ The spectrum contains double lines of the elements hydrogen, helium, carbon and calcium, typical of a normal B5 star. All the lines are diffuse and faint, lacking in contrast. The K lines oscillate in harmony with the other stellar lines. Recently Struve § has detected the presence of a weak interstellar line of stationary position between the oscillating calcium components. His observation has been confirmed by an inspection of our plates.

Four additional examples of this coexistence have been found by the writer. In the two double-lined B3k stars, H.D. 29376, vis. mag. 6·89, and H.D. 39698, vis. mag. 5·89, whose spectroscopic orbits and mass ratios have been determined, ‖ faint stellar K lines were observed in addition to strong interstellar lines. On those spectrograms taken near the γ axes the measures of the K lines indicate a blended origin, but on plates taken near the times of maximum separation three components of the K line were observed. Similar results were found for two spectroscopic binaries previously investigated at this institution. A remeasurement of 7 spectrograms of H.D. 25833, type B3k, vis. mag. 6·61, orbit determined by Plaskett,¶ and of 9 spectrograms of H.D. 214240, type B3k, vis. mag. 6·20, orbit by Hill,** showed the presence of oscillating calcium lines for both components in each star, in addition to the interstellar lines.

It is thus evident from these seven examples that interstellar and stellar calcium coexist in the spectra of the Class B stars of types B2 to B5, and may be separately distinguished in those spectroscopic binaries of sufficiently large amplitude and whose distances are sufficiently great to indicate the interstellar absorption, *i.e.* exceeding 400 parsecs.

III. *Interstellar Calcium in Late-type Spectra*

The detection of an interstellar K line in the spectra of stars later than Class A is a difficult observational problem owing to the presence of the stellar calcium, which blends with and obliterates the absorption due to the intervening cosmic cloud. The great width and strength of the winged

* *Publ. D.A.O.*, **1**, 281, 1921.
‡ *Ibid.*, **1**, 138, 1919.
‖ *Publ. D.A.O.*, **6**, No. 9, in press.
** *Ibid.*, **3**, 358, 1926.
† *Idid.*, **6**, No. 6, in press.
§ *Ap. J.*, **6**, 199, 1930.
¶ *Ibid.*, **3**, 184, 1925.

H and K lines in Classes F and G practically precludes the possibility of detecting a relatively faint interstellar K line in the spectra of any of these stars.

The only possibility of showing the coexistence of stellar and interstellar calcium for these classes would be to find a distant giant star with a radial velocity large enough to displace the stellar K line and reveal the narrow interstellar line projected upon the wing of the broad stellar line. This imposes very severe conditions upon the observer. The minimum distance at which an interstellar K line has sufficient intensity to be positively detected is, according to our recent work, some 400 parsecs. For the line to be seen upon the wing of the stellar line it would require an intensity corresponding to a distance of at least 1000 parsecs. A Go giant of absolute magnitude zero at this distance would be of the 10th apparent magnitude, requiring a very long exposure for a dispersion sufficient to differentiate the two lines. Furthermore, this 10^m Go giant must show a velocity shift, orbital or constant, of 250–300 km./sec., a necessary and essential condition, which so restricts the problem that the probability of finding a Class F or G star which exhibits the coexistence of stellar and interstellar calcium is very small indeed.

For stars of Class B8 to A5, however, the conditions are decidedly more promising, all factors uniting to favour the observation of the two K lines. The stellar K lines for these stars are usually narrow, so that a velocity displacement of only 100 km./sec. would suffice to resolve the two K lines completely, a velocity not uncommon among binary stars of this class. Since the interstellar line is outside the wing of the stellar line and seen against the background of the spectrum, a weaker intensity and hence a considerable less distance of the star would be required. Furthermore, the increased luminosity of these earlier type stars increases the apparent magnitudes, so that our search is among the seventh to ninth magnitude Classes B8 to A5 stars having radial velocities exceeding 100 km./sec. So stringent, however, are the conditions that only one spectrum later than type B5, that of H.D. 698, is known which shows the double calcium lines of different origin.

IV. *The Spectroscopic Orbit of H.D.* 698

This star in Cassiopeiæ, $\alpha = 0^h\ 06^m.3$, $\delta = +57^\circ\ 39'$ (1900), vis. mag. 7·08, spectral Class B9sek, was observed by Merrill * in his survey of Class B emission stars in 1920–24. He found the Hα line to be a strong emission line and classified the spectrum as B8seα. Four spectrograms in the ordinary blue-violet region were obtained, which gave a range of 69 km./sec. in radial velocity, and accordingly its binary nature was announced.

A total of 40 spectrograms have been obtained at Victoria during the past six years. The earlier plates were obtained with the short-focus camera, having a linear dispersion of 49 A. per mm. at Hγ, but the later observations, taken near the descending node to show the triple calcium lines, were obtained with the medium-focus camera, having a dispersion of 29 A. per mm.

* *Ap. J.*, **61**, 389, 1925.

The spectrum of the primary star is typical of Class B9 stars of high luminosity, containing, in addition to a strong helium spectrum, a well-developed metallic spectrum, usually associated with early Class A stars. The relative intensities of the λλ 4481/4471 lines assigns the star to subclass B9. All the lines are narrow and strong, of excellent quality for measurement. The helium and metallic lines undoubtedly arise in the same atmosphere as is shown by the orbital motions of the star. The abnormal association of a helium spectrum with a metallic one, combined with the "*c*" quality of the lines, and the emission character of Hα and Hβ all point to a star of high intrinsic luminosity for Class B9.

The secondary spectrum is faint relative to the primary, and only measureable at times of maximum separation of the lines. It is a normal B5 spectrum, as near as can be estimated, containing only the usual lines of helium I, hydrogen, magnesium II, carbon II, and calcium II of this subclass. It was measured on 24 spectrograms.

Portions of the last two spectrograms obtained are illustrated in Plate 16. In (*a*) the double lines of the *Mg* II/*He* I pairs, λλ 4481/4471, are shown. The camera used was the 1M, dispersion 34 A. per mm. at this region, with a slit width of 0·0015 inch, exposure 2 hours, emulsion Eastman 40. Spectrogram (*b*) was obtained with the 1L camera, dispersion 14 A. per mm. at the K region, slit width 0·002 inch, exposure 3 hours 22 minutes on a Barnet plate. The three calcium lines clearly exhibit the coexistence of stellar and interstellar calcium in this interesting spectrum. The central sharp line is due to interstellar absorption, the strong violet component is the stellar K line of the primary star, while the faint redward component is the stellar K line of the secondary star. Compare this illustration with the radial velocity curves, fig. 2, on which stellar calcium velocities are shown as filled circles.

The orbital elements were determined from 30 observations of the primary star alone, owing to the relatively high probable error of the secondary observations.

Orbital Elements and Minimum Masses of H.D. 698

Period	$P =$	55·904 days.
Eccentricity	$e =$	0·033 ± 0·007.
Periastron passage	$T =$	J.D. 2425634·780.
Longitude of apse	$\omega =$	313°·3 ± 0°·41.
Semi-amplitude, primary	$K_1 =$	85 ·5 ± 0 ·46 km./sec.
Semi-amplitude, secondary	$K_2 =$	215 ·5 ± 3 ·0 km./sec.
Velocity of system	$\gamma =$	−24 ·5 ± 0 ·4 km./sec.
Velocity of calcium Ca^+	=	−13 ·9 ± 0 ·6 km./sec.
$a_1 \sin i$	=	65,700,000 km.
$a_2 \sin i$	=	165,600,000 km.
$m_1 \sin^3 i$	=	113·2⊙, $\frac{m_2}{m_1} = 0{\cdot}40$.
$m_2 \sin^3 i$	=	44·9⊙

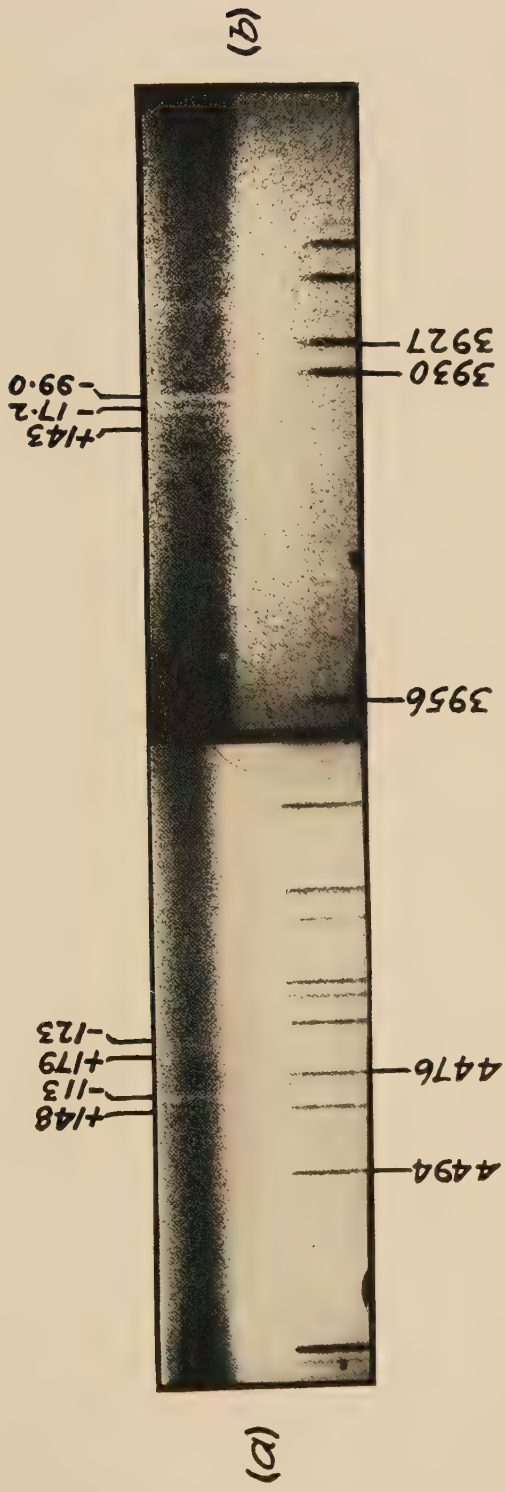

Fig. 1.—Spectra of H.D. 698.

(a) G.C.T. 1931 Oct. 10·288. Phase 39·640.
Obs. Vel. I. −104·2 km./sec.; II. +172·6 km./sec. Red. ⊙ = +4·5 km./sec.
(b) G.C.T. 1931 Oct. 11·370. Phase 40·722.
Obs. Vel. I. −95·6 km./sec.; II. +163·4 km./sec. Red. ⊙ = +4·0 km./sec.

The detailed investigation of this interesting system will appear in the Publications of the Dominion Astrophysical Observatory, and in this section it is only desirable to refer to the unusual masses of the stars. Of those

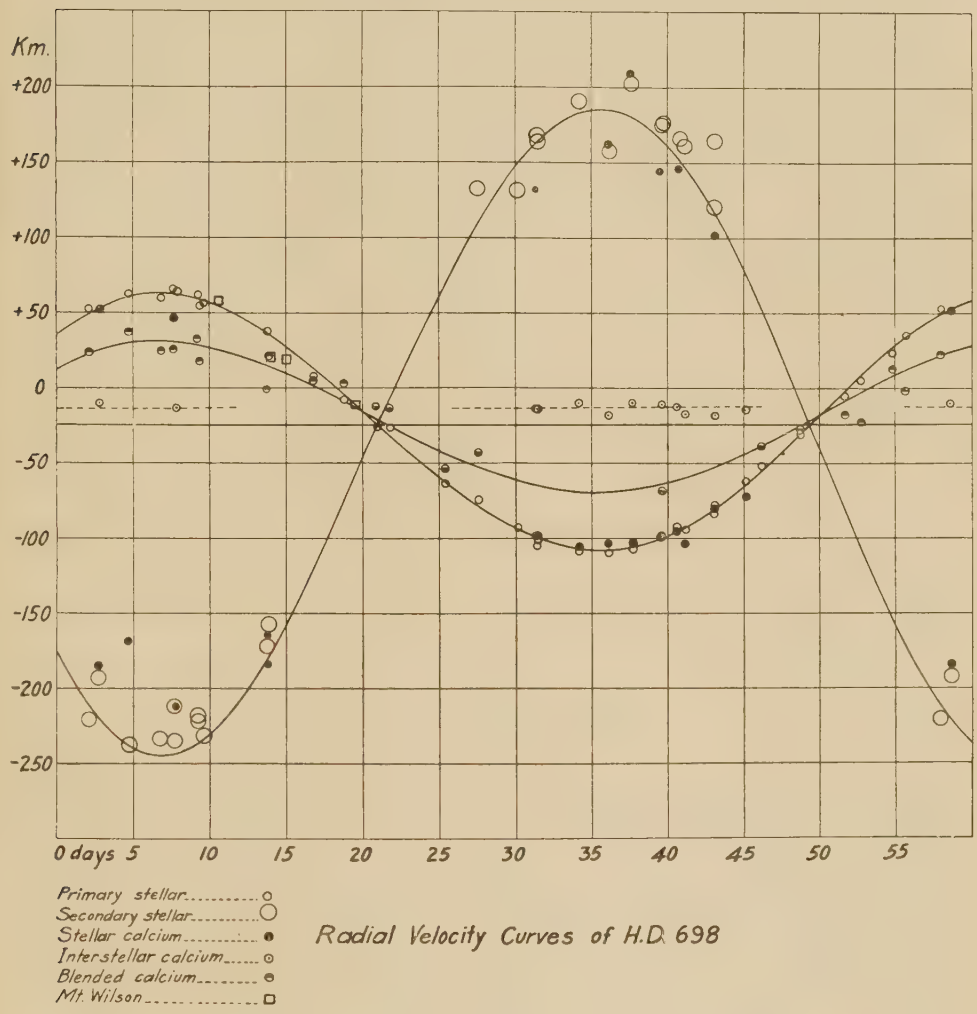

FIG. 2.

binary systems whose orbital systems have been definitely established, this remarkable system, H.D. 698, with a minimum mass, $M \sin^3 i = 158{\cdot}1\odot$, is the most massive thus far discovered.

Next in order are :

Star	Type	$M \sin^3 i$	Investigator
H.D. 47129	O8ek	$138{\cdot}9\odot$	J. S. Plaskett
H.D. 1337	O8nk	$70{\cdot}1\odot$	J. A. Pearce
H.D. 57060	O7sfk	$56{\cdot}5\odot$	J. A. Pearce

V. *The Calcium Lines in H.D.* 698

The most interesting feature of this system is the unique behaviour of the calcium lines. A single K line was measured on 18 of the 40 plates. An inspection of the velocity diagram, fig. 2, will show that from a phase of 46 days to 31 days, *i.e.* for 41 days out of the 56-day period, these measures follow a curve unmistakably indicating a blend of a stellar K line of the primary star with another line of constant velocity. The blended velocities vary from +38 km./sec. to −52 km./sec. about the constant axis of −13·9 km./sec. For a few days during the two-month period, when the stars are passing through the nodes and have maximum relative velocities, this blended line divides into three components as illustrated in fig. 1 (*b*). Eleven secondary calcium, twelve primary calcium and twelve interstellar calcium lines were observed on fifteen spectrograms taken at these favourable epochs. All the observations are shown on the velocity diagram, in which the origin of the various calcium components, whether blended, stellar or interstellar, are clearly indicated.

The twelve observations of the interstellar lines give a velocity of −13·9 ± 0·6 km./sec. The line is narrower than the primary stellar K and has an estimated intensity of 0·6 that of the stellar K line. This intensity ratio would indeed be expected from the relative displacements of the primary star and the blended curve about the interstellar calcium axis, the former being 1·7 times the latter, indicating the relative intensities of the stellar and interstellar lines.

The absolute magnitude of the star may be estimated from the intensity of the interstellar calcium line by a method recently employed by J. S. Plaskett and the writer in our investigation of the interstellar calcium lines in the O5 to B3 stars, the results being in press. The intensity of the line in H.D. 698 is 6·2 on our scale, corresponding to a parallax of 1220 parsecs. This leads to an absolute magnitude of −3·4 for the system, or −3·1 for the B9 component and −1·6 for the B5 component. The stars are outside the region considered by Eddington's Mass-Luminosity relation, and it does not seem justifiable to extrapolate the curve to find their theoretical absolute magnitude. It would be anticipated, however, that such large masses would have greater luminosities than we have found, and it would seem to be difficult to interpret their masses upon the luminosity theory.

Summary

The theory of a uniform distribution of diffuse matter in interstellar space as proposed by Eddington requires that the interstellar lines of calcium and sodium be superimposed upon the spectra of all stars more distant than 400 parsecs, irrespective of their spectral class.

The lines have been generally observed in the stars of spectral Classes O5 to B2 fainter than visual magnitude 5·5, in the spectra of Novæ and Wolf-Rayet stars, and in many of the distant stars of Classes B3 to B5, but have not previously been detected in spectra later than Class B5.

The coexistence of stellar and interstellar calcium lines in the spectra of double-lined spectroscopic binaries of Classes B2 to B5 is in harmony with the theory of interstellar matter. The author has recently added four additional examples of these interesting stars to the three previously known.

The spectroscopic orbits and masses of H.D. 698 have been determined. This star with a minimum mass of 158·1⊙ is the most massive system thus far discovered, the primary star of spectral type B9sek having a mass of 113·2⊙, and the secondary star of type B5 a mass of 44·9⊙.

The behaviour of the calcium lines is the most interesting feature of the spectra. For three-quarters of the 56-day period the measures of the single K line indicate that a blend of an oscillating and stationary line is observed. For a few days, when the stars possess maximum relative velocity, this single calcium line divides into three components, two of which follow precisely the orbital oscillations of the component stars, while the third and sharper K line preserves a stationary position.

The coexistence of stellar and interstellar calcium lines in this interesting spectrum confirms the theory of interstellar calcium for Class B stars.

Dominion Astrophysical Observatory,
Victoria, B.C. :
1932 *June* 29.

NOTE ON THE INTERPRETATION OF STELLAR EMISSION BANDS.

C. S. Beals.

In a recent paper Genard * has shown that the emission bands in the spectra of Nova Cygni and Nova Lacertæ are characterised by considerable complexity of structure, and he has interpreted this complex structure as due to discontinuous and irregular ejection of material from the stellar surface. This conclusion appears to be contrary to the theory of emission band contours presented by the writer,† in which certain approximately flat-topped bands in the spectra of Nova Aquilæ and Nova Cygni were interpreted in terms of a continuous ejection, uniform over the stellar surface.

Admitting the correctness of Genard's interpretation it may still be pointed out that even though areas of more than ordinarily violent ejection may be present it is reasonable to expect that at times there will be approximate spherical symmetry in their distribution over the stellar surface. If certain conditions as to minimum velocities are fulfilled it can be shown that such a condition of approximate spherical symmetry will result in the formation of an approximately flat-topped band. A band formed in this way may exhibit considerable complexity of structure, but this complexity is unlikely to obscure its essentially flat-topped character.

* *M.N.*, **92**, 396, 1932. † *Ibid.*, **91**, 966, 1931.

Genard has raised a point concerning the possibility of photographic over-exposure concealing the true distribution of intensity in a band, and this point is of considerable interest in connection with the writer's previous measures of the bands of Nova Aquilæ and Nova Cygni. These measures were made on plates selected as representing "reasonable" exposures, and after a year's experience in the photometry of carefully standardised spectra the writer is convinced that the densities were not too great for reasonably accurate measurements. The measures published in the paper referred to above were made with a Hartmann visual microphotometer. These measures have since been repeated with a highly accurate photo-electric microphotometer, and the results agree well with the earlier measures. In addition, the band λ4686, which appears in the later stages of Nova Aquilæ, and which is considerably less intense than the hydrogen bands, has also been measured with the photo-electric microphotometer. In this case Hogg's method of employing the intensities of iron arc multiplets in the comparison spectrum has been employed for purposes of calibration. The intensity of the top of the band has been determined as approximately 4·2 times that of the continuous spectrum, a reasonable range to cover on the straight line portion of the characteristic curve. The band shows considerable structure, but there seems no doubt of its essentially flat-topped character.

Genard has suggested that a continuous ejection of atoms, uniform over the stellar surface, may be explained as a result of selective radiation pressure, but that discontinuous and irregular ejection must be due to some other cause. It is, of course, impossible to be certain of the cause of ejection in either case, but there seems no reason why the mechanism of selective radiation pressure should not be adequate to explain both effects. Local variations in temperature over different parts of the stellar surface might perhaps account for the presence of discontinuous and irregular ejection.

Dominion Astrophysical Observatory,
Victoria, B.C.:
1932 *June* 30.

NOTE ON J. M. MOHR'S PAPER "THE ROTATIONAL SPACE MOTIONS OF THE STARS."

Gustaf Strömberg, Ph.D.

In a paper * entitled "The Rotational Space Motions of the Stars" Mr. J. M. Mohr has given very valuable data based on the space motions of galactic stars of known parallax, radial velocity and proper motion. Two different values of the direction of the Sun's velocity around the centre of the galaxy have been used. One attributed to me is in the direction of galactic longitude $L = 71°$; the other, by Oort, has a longitude $L = 55°$. Mr. Mohr gives evidence that the latter value is probably more correct.

* *M.N.*, **92**, 562, 1932.

I wish to call attention to the fact that the value $L = 71°$ derived in *Mount Wilson Contributions*, No. 292,* was only one of a number of determinations. It is simply the Sun's motion referred to 18 globular clusters of known radial velocity, and is, of course, very uncertain. The final value for the Sun's velocity around the centre of the galaxy is the one published at the same time in *Mount Wilson Contributions*, No. 293,† which has the direction $L = 60°$, $B = +10°$, and which within the uncertainties is the same as that derived by other authors. The direction of the axis of asymmetry is $L = 61°{\cdot}5$, $B = +9°{\cdot}0$, and is presumably the direction of motion of a star moving in a circle around the centre of the galaxy. Hence the galactic longitude of this centre is $331°{\cdot}5$, which agrees with other estimates.

One extremely important question could be settled by Mohr's unpublished data. Different opinions have been expressed about the existence of a discontinuity in the velocity distribution for a value of about 70 km./sec. relative to the Sun and in the direction of $L = 240°$. If the asymmetry is due to the non-existence of velocities greater than that of escape *from a local system*, the presence of such a break would support this hypothesis. The question has been discussed in *The Observatory*, 1926 September and October, and in *Mount Wilson Contributions*, No. 332.‡ Mohr's data would easily settle this question if use is made of velocities of apparently bright stars, not selected on the basis of large proper motions, provided the distributions are properly corrected for the effects of accidental errors in the observational data. The necessary formulæ for such corrections have been derived for error-curves by Oort, and for more general types of distribution curves by the writer.‡

The answer to this question is very important for our understanding of the dynamics of galactic rotation.

Carnegie Institution of Washington,
Mount Wilson Observatory :
1932 *June.*

ON THE ORIGIN OF THE SOLAR SYSTEM.

Harold Jeffreys, F.R.S.

A recent article by Prof. F. Nölke § on the origin and development of the solar system seems to call for some comment from me. Much of it is interesting, but the author offers a number of criticisms of the views advanced in recent papers of mine, some of which require replies, while others seem to be based on misunderstandings.

* *Astrophysical Journal*, **61**, 353, 1925.
† *Ibid.*, **61**, 363, 1925.
‡ *Ibid.*, **65**, 238, 1927.
§ Gutenberg's *Handbuch der Geophysik*, **1**, 8–68, 1931.

On p. 29 Nölke says: "An approach to Neptune's distance happens on an average every 3×10^{12} years, to Mercury's distance every 10^{15} years, a collision with the Sun every 10^{17} years. Since the maximum time of development of the Sun, according to Eddington, is of the order of 10^{13} years, it follows that the times required by Jeans, Jeffreys, Moulton and Arrhenius for the necessary nearness of the star exceed quite disproportionately the age of the Sun. According to them, therefore, the development of our system must be held to be a rare exceptional case in cosmical evolution. This is evidently a defect." This argument has already been dealt with in *The Earth*, Appendix B, but as it has frequently been revived some further comment is needed. If we take the number of stars in the universe as 10^9, the above estimate would imply that some one of them has a collision every 10^8 years, and therefore that about 20 solar systems have been formed since ours. The argument is in fact a fallacious application of probability. The task of science is to co-ordinate observed data and predict observable ones. The observed datum here is the existence of one solar system. Other solar systems, whether they exist or not, are unobservable, and it is no task of science to explain the unknown and unknowable. Prof. Nölke's objection is based on the belief that a large fraction of the stars have attendant planetary systems, and this is a pure speculation.

On p. 32, again, we have: "How are we to explain further the fact that the mighty Jupiter was forced by the resistance of the medium into a nearly circular orbit, while the little Mercury, although it was much longer influenced by the medium around the Sun, kept its great orbital eccentricity?" There is no mention of the explanation offered in *The Earth*, § 4·6, that the resistance is not only to the motion of a planet itself, but to that of the condensation around it produced by its attraction. It was shown that on this ground the eccentricity of Jupiter might be expected to decrease at least nine times as fast as that of Mercury.

Nölke proceeds: "Jeans makes no attempt to explain the rotation of the planets. According to Jeffreys they were set into rotation by satellite masses that fell back upon them. In order to bring to the planets their great rotational moment, 1400 Moons with the mass of Jupiter III and 800 Moons with the mass of Titan would have had to unite with their primaries. The improbability of this supposition is evident. Jeffreys hence recently supposes that the planets arose from masses that a foreign star tore away from the Sun, not through tidal action, but by a sideways collision, and which were set into rotational motion through shearing forces. It is naturally possible to explain the rotational movements of the planets on the basis of this and other special hypotheses on the progress of the shock." This passage contains a series of misunderstandings. The use of "recently" (*neuerdings*) suggests that there was a considerable interval of time between the application of the test and the production of a new theory that will account for rotation; in fact, the test and the alternative theory were given in the same paper.* There are no special hypotheses in the collision theory. The collision is supposed to be an ordinary one, that is, not in the line of centres, and not

* *M.N.*, **89**, 636–641, 1929.

a grazing impact ; and the suppositions were what anybody with a knowledge of hydrodynamics would make.

On the same page is a statement that I suppose the effect of a resisting medium on the inclination of satellite orbits to be produced in a few years. I do not recognize this opinion as mine, and I have certainly produced no quantitative estimates of the effects on satellites, beyond pointing out the nature of the difficulties in determining the motion of the medium in the neighbourhood of a planet. Nölke also attributes to me (p. 51) the opinion that the satellites originated and had their eccentricities and inclinations reduced in the distended planetary atmospheres. It is correct that I inferred that the planets would originally expand to radii comparable with the distances of many (not all) of their satellites ; * but I also inferred that the satellites had an individuality as soon as the planets ; and I certainly never expected that the planets maintained these distensions long enough to regularize the satellite's orbits. I may perhaps call attention now to what I omitted to mention previously, that the early distensions are comparable with the size of the gravitational condensations in the resisting medium.† This is a pure coincidence, but will need to be taken into account when a quantitative theory of the history of satellites is attempted.

Nölke's final comment on my work is as follows (p. 65): "Jeffreys's expositions on the liquefaction of the planets can be passed over, since they rest on the suppositions of the Tidal hypothesis, according to which the planetary masses were expelled from the Sun, and consequently in the initial stage of their development had the temperature of the Sun's surface layers (about 3000° C.)." They rest, of course, on the Collision hypothesis, not on the Tidal hypothesis ; and I must assure astronomers that the estimate of the Sun's temperature is not mine.

Since Nölke dismisses the collision hypothesis so summarily, I may be permitted to call attention to its outstanding recommendation, which is shared, so far as I know, by no other hypothesis on the origin (as distinct from the later development) of the solar system. It leads to quantitative results that are in agreement with the facts. It offers direct explanations of the total mass of the planets, the rate of rotation of the Sun, and the total rotational angular momentum of the planets. I do not think that any other theory attempts to do more than to avoid obvious inconsistencies. On the other hand I do not claim that it is completely analysed, and there are many places where opportunities for more precise investigation will suggest themselves to the careful reader.

Nölke's objection concerning the eccentricities of the orbits of the planets, though the form he actually gives it has already been answered, can be restated so as to constitute a real problem, if for Jupiter we substitute Mars, and for Mercury the asteroids. I have already suggested ‡ that the large eccentricities and inclinations of the asteroids might be explained if they were the result of the disruption of a primitive planet when the resisting medium

* *M.N.*, 89, 731–738, 1929.
† *The Earth*, 4·6.
‡ *Loc. cit.*, 4·7.

had almost disappeared. On the other hand a recent suggestion of Prof. E. W. Brown * requires serious attention. The existing theory of perturbations implies that the mean distances of the planets hardly change at all, while the eccentricities and inclinations oscillate in long periods with determinate small amplitudes. Brown points out that the method of treatment is not valid for intervals of time of more than 10^6 or 10^7 years, and suggests that owing to purely gravitational effects the earth's eccentricity may have been reduced from a large value in the last 10^8 years. In that case the chief reason for postulating a resisting medium disappears. Further, if any kind of equipartition of energy is approached, the orbits of the asteroids would be explained. So far there is no direct proof that such changes would actually occur; but supposing it to be obtained, two possibilities arise: first, that the asteroids might have existed as such before the disappearance of the medium, and have acquired their present eccentricities and inclinations later; and second, even that the resisting medium is unnecessary to explain the nearly circular orbits of the greater planets. On the other hand the tests of the theory of the resisting medium already carried out seem to provide a good deal of evidence for its reality.

Nölke indeed argues that before a resisting medium could have produced any important reduction of the eccentricity of Mars it would have brought Phobos down on to the surface of the planet; he offers similar arguments for the other satellites. But he supposes the motion of the medium to be unaffected by the attraction of the planets, which is certainly wrong. I think it is possible that the actual motion near a planet may approximate to a variational orbit, though an exact variational orbit is out of the question.† In that case the effect on the mean distances of satellites would be much smaller; there is evidence that there is some effect, but it is provided by the satellites themselves and by Saturn's rings.‡ The satellites of Mars indeed are difficult objects to explain on any theory. I have previously suggested that they may be captured asteroids, and this hypothesis has now been strengthened by the discovery of a number of asteroids with mean distances less than that of Mars.

The evolution of the medium would result from the outward transfer of angular momentum by viscosity, the inner parts therefore losing as much angular momentum as the outer gained. But since the angular momentum per unit mass increases like the square root of the distance, it follows that the mass driven off to a great distance was a small fraction of the whole: the rest was reabsorbed into the Sun. This consideration, which I had overlooked, is advanced by Nölke on p. 31. It implies that the reabsorption would bring to the Sun angular momentum equal to that of a particle revolving near the Sun's equator and having a mass equal to that of the medium. This, as I showed,§ would be comparable with that of Jupiter. Now Jupiter revolving at such a distance would have about $\frac{1}{\sqrt{5}}$ of its actual angular

* *U.S. Nat. Res. Council*, Bulletin 80, Part V, 1931.
† *The Earth*, 4·34.
‡ *Loc. cit.*, 4·4 and 4·41.
§ *Loc. cit.*, 4·51.

momentum, or about $\frac{2}{3}$ of the rotational angular momentum of the Sun. Thus the reabsorption of the medium accounts for a considerable fraction of the Sun's rotation. Nölke infers that the supply would be much too great; but he has overlooked my estimate of the total mass of the medium. He supposes that the mass of the medium would much exceed the total mass of the planets, in order to produce an important effect on them. But this can be seen to be unnecessary by a simple illustration: a pendulum in an enclosed chamber has its oscillations damped out, even though the enclosed air has a smaller mass than the pendulum. The actual process would be that the planet moving through the medium sends out a conical wave like that due to a rifle bullet. This is ultimately dissipated by viscosity, and its energy is converted into heat and radiated away, the medium continually reapproaching its steady state. Thus the same mass of the medium influences the planets again and again.

Nölke's own theory of the origin of the system is that it began as a nebular arm, which gave the Sun and planets by direct condensation; and he attributes most of the features of the planets to their having been there to begin with. It seems to me that this is on the same footing as the heterogeneous nebula that I considered seriously in 1917,* but soon abandoned. It is not a state that could have persisted long, and immediately suggests the question, "What was the system like just before?" So far as I can see, this can only be a stage of a theory, and we would be entitled to argue at once as to whether the arm belonged to an ordinary spiral nebula, or had been produced by tidal disruption or collision. Nölke does suggest that the original nebula was much larger in extent than the present system, and that the planets were drawn in by an increase in the Sun's mass (or a reduction in its radiation pressure); but it seems clear that neither of these things can have happened during the existence of the earth,† for there has not been time for the Sun's mass to change by more than 1 part in 2000, and therefore the Sun must have been in its present state from the start.

Nölke gives a theory of the formation of satellites. It rests on the Laplace-Roche theory of disruption by rotation; the gaseous matter abandoned round the edge of the lens proceeds to condense, not through mutual gravitational attraction, which is known to be impossible,‡ but through change to the solid or liquid state. The obvious difficulty is in the supposition that the planets ever had enough rotational angular momentum to give disruption of the lenticular type. But the last word has not been said about direct condensation. It could proceed if the density of a constituent in the medium reached its saturation density at the actual temperature. Both are very small, but it remains to be seen which is the smaller and how rapidly condensation could proceed.

* *Science Progress*, No. 45, 52–62, 1917 July.
† *M.N.*, 85, 413–416, 1925.
‡ *Ibid.*, 78, 424–425, 1918.

ON THE NATURE OF DEPENDENCES IN THE REDUCTION OF ASTROGRAPHIC PLATES.

H. C. Plummer, F.R.S.

1. The reduction of astronomical photographs involves problems of two rather different kinds. In the first and more familiar form, when a catalogue was the object, the plates were few, often no more than one for a particular field, and the stars were many. Turner's solution of this problem is in principle perhaps the most generally familiar. In the other form, which is illustrated by parallax work, the relevant stars are few and the plates are many. Though the problem is the same in essence, the practical conditions are slightly different, and an admirable solution was given by Schlesinger which is now generally adopted.

2. A series of plates has been taken of a certain field containing one object star and a limited number of comparison stars. It is assumed that the plates are all well centred (parallel), so that the standard co-ordinates $(X_i,\ Y_i)$ of the comparison stars and $(X,\ Y)$ of the object star are known constants for the series. The latter, however, become $(X+\Delta X,\ Y+\Delta Y)$ for a particular plate; it will be supposed that the determination of ΔX is the object of the reduction. The corresponding measures on a plate are denoted by $(x_i,\ y_i)$ and by $(x,\ y)$. The field is so restricted that a linear solution is justified.

Consider first the case in which the number of comparison stars is reduced to the minimum, three. Then

$$\begin{array}{l|l} x_1 - X_1 = aX_1 + bY_1 + c & D_1 \\ x_2 - X_2 = aX_2 + bY_2 + c & D_2 \\ x_3 - X_3 = aX_3 + bY_3 + c & D_3 \end{array}$$

and ($a\Delta X$, $b\Delta Y$ being made negligible by proper adjustment)

$$x - (X + \Delta X) = aX + bY + c.$$

One method of conducting the solution for ΔX, quite appropriate for a single plate, would be to solve the first three equations for a, b, c and to substitute the resulting values in the fourth. But clearly it is a simple question of elimination; the desired result could be immediately written down in determinant form. The most practical mode of effecting the elimination is by introducing the multipliers D_1, D_2, D_3, such that

$$\left.\begin{array}{l} D_1X_1 + D_2X_2 + D_3X_3 = X \\ D_1Y_1 + D_2Y_2 + D_3Y_3 = Y \\ D_1 \quad + D_2 \quad + D_3 \quad = 1 \end{array}\right\}, \tag{1}$$

and then

$$D_1(x_1 - X_1) + D_2(x_2 - X_2) + D_3(x_3 - X_3) = x - (X + \Delta X)$$

or $$\Delta X = x - X - D_1(x_1 - X_1) - D_2(x_2 - X_2) - D_3(x_3 - X_3). \qquad (2)$$

The advantage of this method is, of course, that the dependences D_1, D_2, D_3 are thus determined from the standard co-ordinates alone and, once determined, apply to the whole series of plates.

3. The meaning of these dependences is easily seen in geometrical terms. The solution of (1) gives immediately

$$\frac{D_1}{\begin{vmatrix} X & X_2 & X_3 \\ Y & Y_2 & Y_3 \\ 1 & 1 & 1 \end{vmatrix}} = \frac{D_2}{\begin{vmatrix} X_1 & X & X_3 \\ Y_1 & Y & Y_3 \\ 1 & 1 & 1 \end{vmatrix}} = \frac{D_3}{\begin{vmatrix} X_1 & X_2 & X \\ Y_1 & Y_2 & Y \\ 1 & 1 & 1 \end{vmatrix}} = \frac{1}{\begin{vmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ 1 & 1 & 1 \end{vmatrix}}$$

or, if S_1, S_2, S_3 are the three comparison stars, and P is the object star,

$$\frac{D_1}{PS_2S_3} = \frac{D_2}{PS_3S_1} = \frac{D_3}{PS_1S_2} = \frac{1}{S_1S_2S_3}$$

expresses the dependences as the ratios of the triangles formed by the four stars.*

One point may be noticed before leaving the case of three comparison stars. Since

$$(D_2 - D_3)^2 + (D_3 - D_1)^2 + (D_1 - D_2)^2 = 3(D_1^2 + D_2^2 + D_3^2) - (D_1 + D_2 + D_3)^2,$$

or $$D_1^2 + D_2^2 + D_3^2 = \tfrac{1}{3}\{(D_2 - D_3)^2 + (D_3 - D_1)^2 + (D_1 - D_2)^2 + 1\},$$

it is evident that $D_1^2 + D_2^2 + D_3^2$ is a minimum when $D_1 = D_2 = D_3 = \frac{1}{3}$. This is the condition which reduces the probable error of ΔX, by (2), to a minimum. It requires the equality of the triangles PS_2S_3, PS_3S_1, PS_1S_2, and this means that the object star P coincides with the centroid of the triangle S_1, S_2, S_3.

4. When the number of comparison stars (n) exceeds the minimum, the plate constants are determined by the normal equations

$$\left.\begin{aligned} a[X_i^2] + b[X_iY_i] + c[X_i] &= [X_i(x_i - X_i)] \\ a[X_iY_i] + b[Y_i^2] + c[Y_i] &= [Y_i(x_i - X_i)] \\ a[X_i] + b[Y_i] + nc &= [x_i - X_i] \end{aligned}\right| \begin{aligned} A \\ B \\ C \end{aligned}$$

while the equation

$$aX + bY + c = x - (X + \Delta X)$$

remains as before. To effect the elimination of a, b, c by factors independent of the measured co-ordinates, let

$$\left.\begin{aligned} A[X_i^2] + B[X_iY_i] + C[X_i] &= X \\ A[X_iY_i] + B[Y_i^2] + C[Y_i] &= Y \\ A[X_i] + B[Y_i] + Cn &= 1 \end{aligned}\right\}, \qquad (3)$$

* Schlesinger (*A.J.*, **37**, 874, 1926) has given an unsymmetrical representation of the three dependences from which this symmetrical form can be immediately deduced. The same geometrical relations are employed in a note by S. Arend (*C.R.*, **194**, 2292, 1932).

and then

$$[(AX_i+BY_i+C)(x_i-X_i)]=x-X-\Delta X.$$

If therefore

$$AX_i+BY_i+C=D_i, \qquad (i=1, 2, \ldots, n) \tag{4}$$

it will follow that

$$\Delta X=(x-X)-\Sigma D_i(x_i-X_i), \tag{5}$$

where the dependences D_i are determined for the whole series of plates by a single process.

The third equation of (3) shows that $\Sigma D_i=1$. Now

$$\Sigma(D_i-D_j)^2=n\Sigma D_i^2-(\Sigma D_i)^2$$

or

$$\Sigma D_i^2=\frac{1}{n}\{\Sigma(D_i-D_j)^2+1\}.$$

Hence ΣD_i^2 is a minimum when all the dependences D_i are equal, and this is the condition, by (5), which renders the probable error of ΔX a minimum.

5. In seeking a geometrical representation of the dependences in this case, the origin of co-ordinates is clearly not material. It can be identified with the centroid of the comparison stars treated as points of equal mass. Then $[X_i]=[Y_i]=0$, $C=1/n$, and

$$\begin{aligned} A[X_i^2] \quad &+B[X_iY_i]=X \\ A[X_iY_i]&+B[Y_i^2] \quad =Y. \end{aligned}$$

Further simplification might be made by identifying the axes with the principal axes of the same mass system, thus making $[X_iY_i]=0$. But the direction of the measured co-ordinate will be retained. Thus $C=n^{-1}$ and

$$\begin{aligned} A\{[X_i^2][Y_i^2]-[X_iY_i]^2\}&=X[Y_i^2]-Y[X_iY_i] \\ B\{[X_i^2][Y_i^2]-[X_iY_i]^2\}&=Y[X_i^2]-X[X_iY_i]. \end{aligned}$$

Now the dependences, given by (4), are proportional to the perpendiculars from (X_i, Y_i) to the line

$$A\xi+B\eta+C=0,$$

or

$$\xi\{X[Y_i^2]-Y[X_iY_i]\}+\eta\{Y[X_i^2]-X[X_iY_i]\}+n^{-1}\{[X_i^2][Y_i^2]-[X_iY_i]^2\}=0.$$

This line is the polar of the point $(-X, -Y)$ with respect to the quadric

$$\xi^2[Y_i^2]-2\xi\eta[X_iY_i]+\eta^2[X_i^2]=n^{-1}\{[X_i^2][Y_i^2]-[X_iY_i]^2\}, \tag{6}$$

and this quadric is, in a precise sense, the momental ellipse of the mass system defined by the comparison stars.

Hence the geometrical representation of the dependences becomes apparent. Let G be the centroid of unit masses at the points S_i, and Q the equidistant point diametrically opposite the object $P(PG=GQ)$. The momental ellipse (6) is drawn; had the axes been taken parallel to the principal axes its equation would be

$$\frac{\xi^2}{[X_i^2]}+\frac{\eta^2}{[Y_i^2]}=\frac{1}{n}.$$

Then the dependences D_i, whose sum is always unity, are proportional to the perpendiculars drawn from the corresponding points S_i to the polar of Q with respect to this momental ellipse.

This representation involves a proof of the familiar configuration required in well-placed comparison stars. It has been seen that the values of the dependences must be equal if the probable error in the deduced value of ΔX is to be a minimum. Therefore the perpendiculars from S_i on the polar of Q must all be equal. But this is in general impossible unless the polar is at infinity. Hence Q, and at the same time the object P, must coincide with G, which is the centre of the momental ellipse and the centroid of the comparison stars.

6. The geometrical representation of the dependences D_i has taken different forms according as the number of comparison stars is the least possible or in excess of that number. When it is sought to reconcile them by reducing the more general to the particular case, the convenience of a triangular system of co-ordinates, as opposed to the Cartesian co-ordinates of practical measurement, quickly appears. The results are most simply expressed in areal co-ordinates for the triangle of reference $S_1S_2S_3$.

The points opposite S_1, S_2, S_3 in the centroid G being denoted by S_1', S_2', S_3', the polars of these points with respect to the momental ellipse at G must be the sides of the triangle $S_1S_2S_3$, for when the object point coincides with S_1, it is obvious that $D_2 = D_3 = 0$ and the polar passes through S_2, S_3. The co-ordinates of S_1', S_2', S_3' are $(-1, 2, 2)$, $(2, -1, 2)$ and $(2, 2, -1)$. Hence it is easily found that

$$x^2+y^2+z^2-4yz-4zx-4xy=0$$

is the equation of the momental ellipse required. If the object point P is (x_0, y_0, z_0) and the opposite point Q is (x', y', z'),

$$\frac{x_0}{2y'+2z'-x'}=\frac{y_0}{2z'+2x'-y'}=\frac{z_0}{2x'+2y'-z'}.$$

The polar of Q thus becomes the line

$$xx_0+yy_0+zz_0=0,$$

and it is at once seen that the perpendiculars on this line from S_1, S_2, S_3 are proportional to x_0, y_0, z_0. Thereby the result for the special case is deduced from the more general representation.

The economy of labour introduced by Schlesinger's method of dependences is so remarkable from a practical point of view that any attempt to extend the application of this beautiful method should be welcome. Its scope is not limited to the special problem originally contemplated and chiefly considered in the present note.

Erratum.

P. 839, 4th line from bottom of page, *for* 0·002 inch *read* 0·0002 inch.

LIST OF ADDITIONS

TO THE

LIBRARY OF THE SOCIETY

1931 JUNE 30 TO 1932 JUNE 30

Serial Publications received by exchange in the regular course, and Reprints of Papers published in some of these Serials, are not included in the Annual List.

An asterisk (*) indicates that the work is an excerpt.

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